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CREOL, the College of Optics and Photonics, Univ. of Central Florida

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Optronic Laboratories

Optronic Laboratories LLC 4632 36th Street, Orlando, FL 32811
Tel: 1 407 422 3171 Fax: 1 407 648 5412 Email: info@olinet.com

Transmission matrices of random media: means for spectral polarimetric measurements

Thomas W. Kohlgraf-Owens and Aristide Dogariu*

CREOL, The College of Optics and Photonics, The University of Central Florida, 4000 Central Florida Boulevard, Orlando, Florida 32816, USA

**Corresponding author: adogariu@creol.ucf.edu*

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The field transmitted through disordered media is essentially a randomly sampled version of the incident field. Properties of the initial field can be recovered if this sampling function or transmission matrix is known. Here we demonstrate how the transmission matrix of a disordered material can be used to simultaneously measure the spectral and polarimetric properties of an optical field. © 2010 Optical Society of America

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Because random media are ubiquitous in the real world, there has been great interest in the transmission matrices (TMs) associated with such materials [1–8]. The full TM of a disordered system is extremely large, because it describes the propagation of fields with any wavelength, polarization, wavefront, and degree of coherence [9]. As a result, disordered materials could be expected to be able to serve in place of different types of traditional optical components or systems if the appropriate subsets of the TM were known [1].

One of the earliest demonstrations of using the TM of a random material was the polarimetric measurement of an incident optical field by correlating the transmitted speckle pattern with several other reference fields [10]. Focusing of acoustic and electromagnetic waves through a random material has also been demonstrated using time reversal of the speckle field generated by a point source, although time reversal should work for more complicated source distributions as well [2,3]. Later experiments showed that time reversing a speckle pattern is not necessary to focus through a random material; noniterative wavefront shaping techniques can be used to find the appropriate wavefront [4].

Recently, many of the predicted uses of random materials with known TMs have been realized experimentally. For example, work has focused on how the TM of a random material varies spectrally or polarimetrically for a fixed wavefront and then has used these properties to perform optical measurements [5–7]. The TM of a random material as a function of incident wavefront at a given wavelength and state of polarization has also been measured and used to focus light into arbitrary patterns [8]. In this Letter, we demonstrate that randomly scattering materials have sufficient diversity in their spectral transmission, both spatially and polarimetrically, to allow precise measurement of the spectrally dependent polarization state of an optical field with a given wavefront.

For a static medium, the wave interferences inside the medium are fixed and, as a result, the TM of the material can be discussed in terms of either intensities or fields. For the sake of making the polarimetric discussion clearer, we will discuss the TM in terms of intensity transmission. A related development concerning transmitted fields has been previously presented [6]. In a static, linear system, the transfer of the Stokes vector is given by a wavelength-dependent 4×4 Mueller matrix, $\mathbf{M}(\mathbf{x}, \lambda)$. If intensity measurements of the scattered light, not polarimetric measurements, are made, only the first row of $\mathbf{M}(\mathbf{x}, \lambda)$, denoted as $\mathbf{M}_1(\mathbf{x}, \lambda)$, is important, and it can be calculated after exposing the medium to reference states of polarization and measuring the transmitted intensities at position $\mathbf{x}$. After finding several $\mathbf{M}_1(\mathbf{x}, \lambda)$'s for many different wavelengths, the spectrally dependent polarimetric properties of an unknown field can be determined by inverting the sampled version of the incident field produced by the known $\mathbf{M}_1(\mathbf{x}, \lambda)$'s.

A complete estimation of the Stokes vector for a particular wavelength can be made by measuring the intensity after passing the light through at least four different $\mathbf{M}_1$'s [11]. The measurement can be expressed as

$$\mathbf{I} = \bar{\mathbf{M}}\mathbf{S}, \tag{1}$$

where $\mathbf{I}$ is a $n \times 1$ row vector, with $n \geq 4$, composed of the intensity measurements through n different $\mathbf{M}_1$'s. The matrix $\bar{\mathbf{M}}$ is the $n \times 4$ TM whose rows are the $\mathbf{M}_1$'s corresponding to the appropriate intensity measurement, and $\mathbf{S}$ is the 4×1 Stokes vector of the incident light. For $n = 4$, $\mathbf{S}$ can be estimated by inverting the TM as $\hat{\mathbf{S}} = \bar{\mathbf{M}}^{-1}\mathbf{I}$. Here $\hat{\mathbf{S}}$ denotes the estimate of the Stokes vector based on the measured intensities. When more than four intensity measurements are used, the linear system given by Eq. (1) becomes inconsistent, so the least-squares solution for the Stokes vector, given by the Morse–Penrose pseudoinverse, is usually found.

The generalization to measure the Stokes vector for multiple wavelengths is straightforward. The measured intensities will be the result of contributions from multiple wavelengths, so the rows of Eq. (1) must be integrated over wavelength:

$$I_i = \int \bar{\mathbf{M}}_i(\lambda)\mathbf{S}(\lambda)\mathrm{d}\lambda. \tag{2}$$

For practical purposes, the system response can be averaged over different spectral bands to produce a discrete approximation of the actual spectrum. The widths of these bands should be small relative to the spectral variations of the unknown field and such that the system's response varies from band to band. This averaging

effectively converts the integral in Eq. (2) to a summation over a finite number of spectral bins:

$$I_i \approx \sum_{\lambda} \bar{\mathbf{M}}_i(\lambda)\mathbf{S}(\lambda). \tag{3}$$

Therefore, the Stokes vectors for the additional wavelengths are simply appended to the original $\mathbf{S}$, making it a $4\ m \times 1$ row vector, where m is the number of spectral bins for which the polarization properties of the light are being estimated. The TM becomes an $n \times 4\ m$ matrix, where n is the number of intensity measurements with $n \geq 4\ m$. The inversion procedure used to estimate the Stokes vector remains unchanged, although constraints must be added to the inversion to ensure that the resulting Stokes vectors are physically realizable.

To test this concept, we used a setup similar to the one described in [7]. One end of a 140 μm diameter imaging fiber bundle with approximately 3 μm cores was covered with a nondiffusing layer of 0.5 μm silica spheres. Because the spheres are located randomly with respect to the centers of the fiber cores, which are inherently asymmetric, different cores have widely varying responses, both spectrally and polarimetrically. The average spectral contrast, i.e., the standard deviation of the spectral response for each core divided by its mean averaged over all of the fiber cores, was approximately 24%. The wavelength averaged polarimetric contrast, defined analogously to the spectral contrast, was approximately 31%. So, the scattering medium used in our experiments has significant variations in both its polarimetric and spectral responses.

The other end of the fiber bundle was imaged onto a CCD camera. Two light sources were used during the experiment as calibration and test sources. The first was an Optronic Laboratories OL 490 light source with adjustable bandwidth and the ability to generate complex spectra. In this case, the calibration spectra for the spectral bins had an FWHM of approximately 8 nm, so that the center 3 nm were approximately uniform in intensity. The bins ran from 520 to 580 nm in 3 nm steps. The second light source was a halogen lamp filtered with an acousto-optic tunable filter (AOTF) with an FWHM of 2.5 nm. It was used for spectral bins running from 544 to 557.5 nm in 1.5 nm steps.

The state of polarization of the incident light was controlled using a polarizer and a broadband, quarter-wave plate. The calibration was carried out in a manner similar to that described in [6]. Four orientations of the polarization optics were used to measure the $\mathbf{M}_1(\mathbf{x}, \lambda)$ of each of the fiber cores. The wavelength of the source was swept over the desired range for each configuration of the polarization optics.

After measuring $\mathbf{M_1}(\mathbf{x}, \lambda)$ for each fiber core, groups of fibers can be formed to measure the spectropolarimetric properties of an unknown field. Because at least four measurements are required per wavelength, a minimum of 80 cores, or 36 cores per group, is required to estimate $\mathbf{S}(\lambda)$ for the two different light sources; however, because each CCD image contains over 1200 fiber cores, the estimation was performed using approximately 700 cores to reduce the effects of experimental noise. The estimation process was repeated many times using randomly selected groups, and the average result was computed.

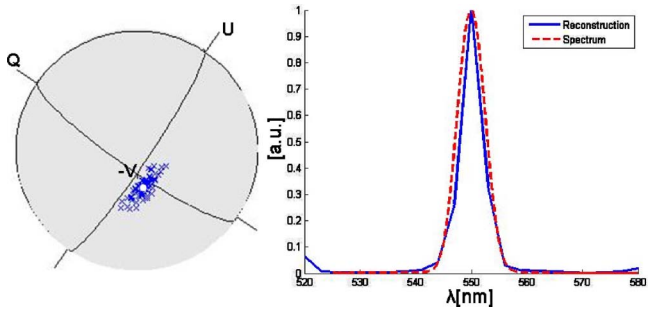

Fig. 1. (Color online) Left panel, Poincaré sphere representation of the measured polarization states using different groups of fiber cores (blue crosses) and the average measured state (white dot) for a $-V$ incident state. Right panel, normalized spectrum recovered using the new scattering-based procedure (Reconstruction) and the input spectrum measured by a typical spectrometer (Spectrum) with 3 nm spectral resolution.

The first test consisted of an optical field characterized by a single spectral peak at 550 nm with an FWHM of 6 nm, and a polarization state $[I, Q, U, V] = [1, 0, 0, -1]$. The results of the reconstruction are shown in Fig. 1. Each blue cross on the Poincaré sphere denotes the state of polarization recovered using a different group of fiber cores; because of inherent experimental noise, the reconstructed states have some spread on the Poincaré sphere. However, the center of mass of their distribution, indicated by the white dot, differs by only 1% from the input state of polarization. The uncertainty in estimating the Stokes vectors along the Q and U axes are better than 0.04 and 0.1, respectively. Testing indicates that the slight asymmetry in the estimates is likely due to chromatic aberration in the imaging optics.

The input and recovered spectra are also shown in Fig. 1. A 3-nm-wide averaging window was applied to the input spectrum to make it comparable to the spectrum reconstructed using our scattering-based procedure. Clearly, the input spectrum is recovered quite well. When a Gaussian profile is fitted to each of them, the FWHMs are 3.6 and 5.8 nm for the reconstructed and incident spectra, respectively. The rms difference between the spectra is only 0.5%.

The new procedure has been applied to even more complex situations. For instance, the reconstruction of input spectra having two and three well-defined modes is illustrated in Fig. 2. In both cases, the input field was in the same polarization state as in the previous example. The errors in the recovered polarization states at the peak wavelengths range between 1% and 1.5%, and the spread in the Q and U components are 0.07 and 0.28. As can be seen, the spectral peaks are well recovered;

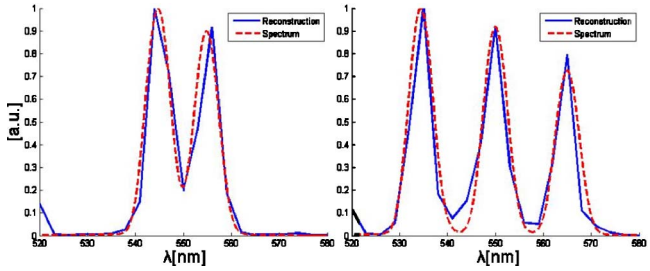

Fig. 2. (Color online) Input and recovered complex spectra. The polarization state was constant across both spectra.

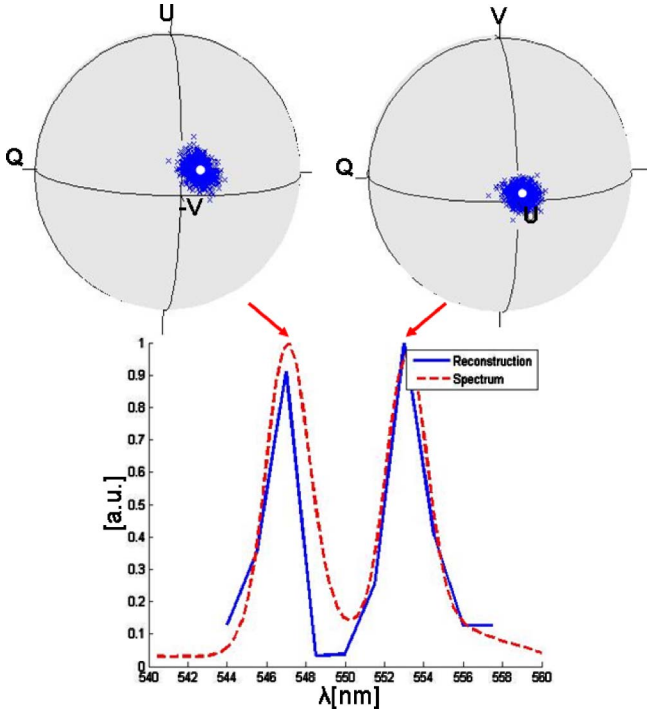

Fig. 3. (Color online) Input and recovered optical fields with spectrally varying states of polarization. The arrows indicate the spectral bin represented on the Poincaré spheres. The color code is the same as in Fig. 1, except that the dashed spectral curve was smoothed with a 1.5-nm-wide window.

the rms differences between the reference and estimated spectra are less than 1.25%

Finally, we demonstrate the ability to recover optical fields with spectrally varying states of polarization. To test the procedure in this case, the random medium was illuminated sequentially with individual spectral peaks, each with different a state of polarization. The images from the individual spectra were then added together to produce a spectrum with well-separated polarization states at different wavelengths. In the example presented in Fig. 3, the test field has a spectrum containing a peak at 547 nm polarized in the $-V$ state and another peak at 553 nm but polarized in the U state. Neither of these polarization states were used as reference states for obtaining the TM. As can be seen in Fig. 3, the agreement between the expected and recovered properties is quite good. The error in recovering the $-V$ polarization state was less than 3%, while, for the U polarization state, the error was less than 1%. The rms difference between the input and recovered spectra is 3.3%. The slightly increased errors for the recovery in Fig. 3 are likely due to mechanical instabilities in our experimental setup. The intensity from the AOTF was significantly less than the OL 490 light source and, as a result, longer integration times were needed to collect the calibration data.

The accuracy of the spectral and polarimetric measurements are comparable to some commercial devices and could be further improved by minimizing the chromatic aberration in the imaging optics and employing more polarization states in the calibration process. The maximum resolution of the spectral recovery is a function of the spectrum of the calibration source, because using a small bin separation with a broad source will result in ambiguity as to which bin certain wavelengths belong.

Accurate measurements of spectra using fiber probes are commonly performed; however, the polarization properties of the light have been largely neglected because the state of polarization is not, in general, maintained upon propagation through an optical fiber. The concept presented here can be extended to measure other properties of the incident field, for example, its wavefront, by utilizing a larger subset of the TM of the material.

Utilizing the TM of a random medium, we have demonstrated here for the first time, to our knowledge, that both the spectral and polarimetric properties of an optical field can be simultaneously recovered by encoding those properties into intensities through a random scattering process. In fact, the potential of TMs of scattering media is only beginning to be explored experimentally. Indeed, as was pointed out nearly two decades ago, the limitations in using a random medium for optical measurements or processing are primarily due to the reference fields used to select elements from the TM [1].

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